KALMAN FILTER

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Abstract: Kalman filtering was developed in the 1960s, although it has its roots as far back as Karl Gauss in 1795. The Kalman filter is an efficient recursive filter that estimates the state of a linear dynamic system from a series of noisy measurements. It is used in a wide range of engineering applications from radar to computer vision, and is an important topic in control theory and control systems engineering.

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1. INTRODUCTION

We have been filtering things from the beginning of our history. For example Water filtering is a simple example, we filter impurities from water simply using our hands to hold dirt and leaves. Another example is filtering out noise from our surroundings. If we paid attention to all the little noises around us we would go crazy. We learn to ignore superfluous sounds and focus on important sounds.

There are also many examples in engineering where filtering is desirable. Radio communications signals are often corrupted with noise. A good filtering algorithm can remove the noise from electromagnetic signals while still retaining the useful information.

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The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state. In contrast to batch estimation techniques, no history of observations and/or estimates is required.

An example application would be providing accurate, continuously updated information about the position and velocity of an object given only a sequence of observations about its position, each of which includes some error. For example, in a radar application where one is interested in tracking a target, information about the location, speed, and acceleration of the target is measured at each time instant with a great deal of corruption by noise. The Kalman filter exploits the trusted model of the dynamics of the target, which describes the kind of movement possible by the target, to remove the effects of the noise and get a good estimate of the location of the target at the present time (filtering), at a future time (prediction), or at a time in the past (interpolation or smoothing).

Alternatively, [1] consider an old slow car that is known to go from 0 to 60 kilometers per hour (km/h) in no less than 10 seconds. The speedometer on this car however shows very noisy measurements that vary wildly within a 40 km/h window around the actual speed of the car. From stop – which is measured with certainty because the wheels are not turning – the driver of the car pushes its gas pedal as far as possible. Five seconds later, the speedometer reads 70 km/h. The driver concludes that the slow car cannot be traveling that quickly and uses information about the known speedometer noise to conclude that the car is likely traveling at 30 km/h instead. Similarly, a Kalman filter uses information about noise and system dynamics to reduce uncertainty from noisy measurements.

2. THE KALMAN FILTER

Consider the problem of estimating the variables of some system. In dynamic systems (that is, systems which vary with time) the system variables are often denoted by the term *state variables*. Assume that the system variables, represented by the vector x, are governed by the equation $x_{k+1}=Ax_k+w_k$ where w_k is random process noise, and the subscripts on the vectors represent the time step. For instance, if our dynamic system consist of a spacecraft which is accelerating with random bursts of gas from its reaction control system thrusters, the vector x might consist of position p and velocity v. Then the system equation would be given by the equation 1[5]:

$$\begin{bmatrix} p_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_k \\ v_k \end{bmatrix} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} a_k \tag{1}$$

where a_k is the random time-varying acceleration, and T is the time between step k and k+1. Now suppose we can measure the position p. Then our measurement at time k can be denoted $z_k = p_k + v_k$ where v_k is random measurement noise.

The question which is addressed by the Kalman filter is this: [5] Given our knowledge of the behavior of the system, and given our measurements, what is the best estimate of position and velocity? We know how the system behaves according to the system equation, and we have measurements of the position, so how can we determine the best estimate of the system variables? Surely we can do better than just take each measurement at its face value, especially if we suspect that we have a lot of measurement noise.

The Kalman filter is formulated as follows. Suppose we assume that the process noise w_k is white with a covariance matrix Q. Further assume that the measurement noise v_k is white with a covariance matrix R, and that it is not correlated with the process noise. We might want to formulate an estimation algorithm such that the following statistical conditions hold:

- The expected value of our state estimate is equal to the expected value of the true state. That is, on average, our estimate of the state will equal the true state.
- We want an estimation algorithm that minimizes the expected value of the square of the estimation error. That is, on average, our algorithm gives the smallest possible estimation error.

It so happens that the Kalman filter [5] is the estimation algorithm which satisfies these criteria. There are many alternative ways to formulate the Kalman filter equations. One of the formulations is given in the equations 2-5 as follows:

$$S_k = P_k + R \tag{2}$$

$$K_k = AP_kS_k^{-1} \tag{3}$$

$$P_{k+1} = AP_kA^T + Q - AP_kS_k^{-1}P_kA^T \tag{4}$$

$$\hat{x}_{k+1} = A\hat{x}_k + K_k(z_{k+1} - A\hat{x}_k) \tag{5}$$

In the above equations, the superscript –1 indicates matrix inversion and the superscript T indicates matrix transposition. S is called the covariance of the innovation, K is called the gain matrix, and P is called the covariance of the prediction error.

Equation 5 is fairly intuitive [5]. The first term used to derive the state estimate at time k+1 is just A times the state estimate at time k. This would be the state estimate if we didn't have a measurement. In other words, the state estimate propagates in time just like the state vector (see Equation 1). The second term in Equation 5 is called the correction term, and it represents how much to correct the propagated estimated due to our measurement. If the measurement noise is much greater than the process noise, K will be small (that is, we won't give much credence to the measurement). If the measurement noise is much smaller than the process noise, K will be large (that is, we will give a lot of credence to the measurement).

3. APPLICATION OF KALMAN FILTERS FOR MECHANICAL SYSTEMS WITH RANDOM BURST OF ACCELERATIONS.

The system represented by Equation 1 [5] was simulated on a computer with random bursts of acceleration which had a standard deviation of 0.15 m/sec^2. The position was measured with an error of 3 m (one standard deviation). The figure below shows how well the Kalman Filter was able to estimate the position, in spite of the large measurement noise.

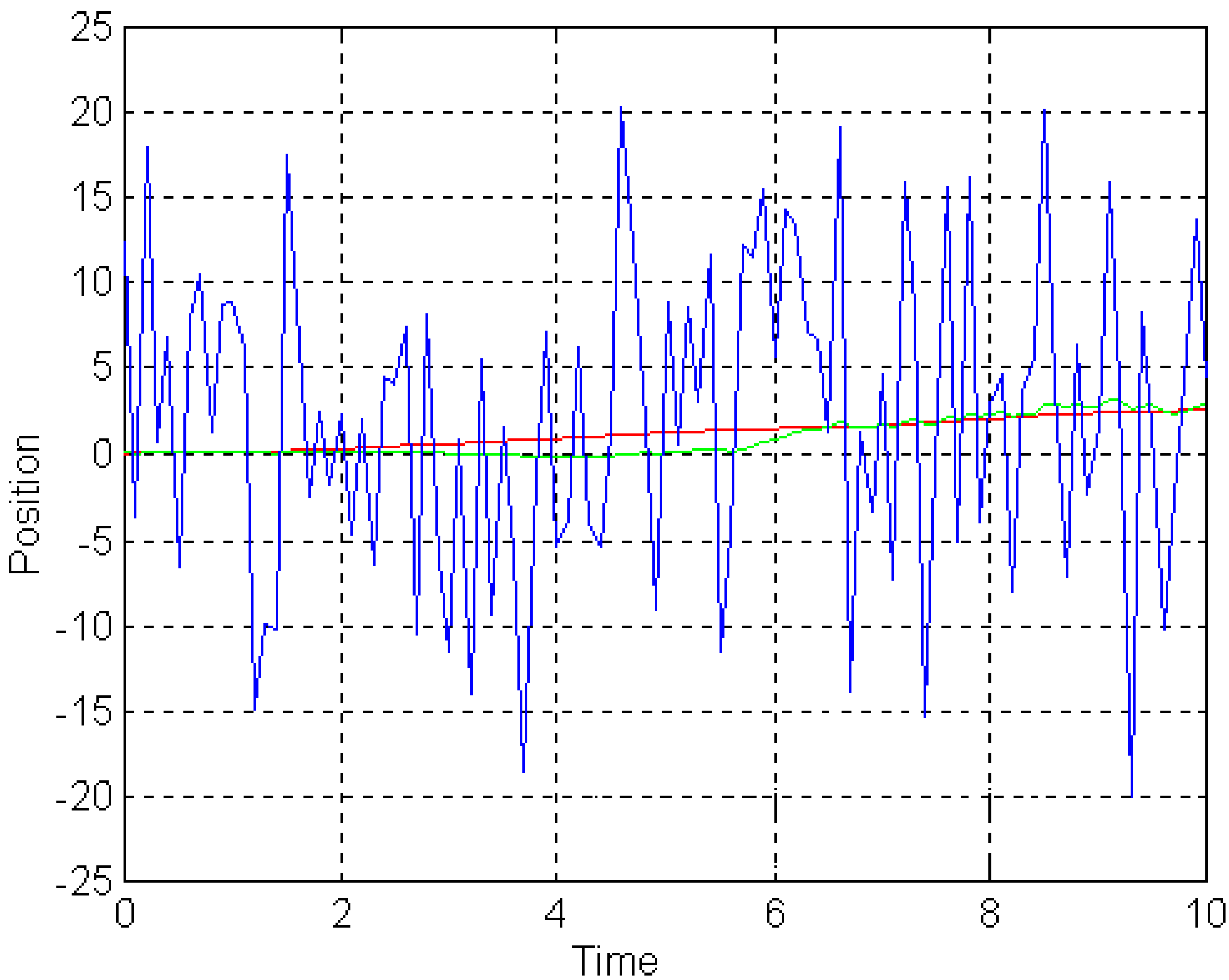

Fig.1. Diagram of true position (red), estimated position (green) and measured position (blue)

4. CONCLUSIONS

Kalman filters are a very efficient and versatile method to filter noisy signals to control actuators based on sensor feedback and also to estimate system evolution in time. In unconventional machine tools (ultrasound and spark processing machines), kalman filters can be used to control feed movements of the tool in order to have smooth variations. Another application for spark processing machine is to control the spark current.

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